1. Suppose T is an unbiased estimator of $g(\theta)$ in a regular model. Which statement is always true?

A. If T is the UMVUE, it achieves the Cramér–Rao lower bound for every θ.

B. If T achieves the Cramér–Rao lower bound for every θ, it is the UMVUE.

C. Every unbiased estimator achieves the Cramér–Rao lower bound for at least one θ.

D. The Cramér–Rao lower bound equals $1/I_{\underline{X}}(\theta)$, regardless of g.

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2. A test of $H_0 : \theta \leq 0$ versus $H_1 : \theta > 0$ has level 0.05. Which statement must be true?

A. The probability that H_0 is true is at most 0.05.

B. The probability that H_1 is true is at least 0.95.

C. For every $\theta \leq 0$, the probability of rejecting H_0 is at most 0.05.

D. For every $\theta > 0$, the probability of rejecting H_0 is at least 0.95.

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3. Let $X \sim \text{Exp}(\theta)$. Which is the UMP level-0.1 test of $H_0 : \theta \leq 1$ versus $H_1 : \theta > 1$ based on X?

Hint: $\mathbb{E}_\theta[X] = 1/\theta$. Should we reject if $X >$ cutoff (large X) or $X <$ cutoff (small X)?

Useful fact: $\mathbb{P}_\theta(X > x) = e^{-\theta x}$ for $x > 0$.

A. Reject when $X > -\ln(0.1)$.

B. Reject when $X < -\ln(0.1)$.

C. Reject when $X > -\ln(0.9)$.

D. Reject when $X < -\ln(0.9)$.